Fracture characterization of short glass fibre reinforced thermoplastic polyester by the *J*-integral

B. H. KIM, H. S. KIM

Department of Mechanical Engineering, University of Massachusetts, Amherst, MA 01003, USA.

Fracture initiation of short glass fibre reinforced thermoplastic polyester was characterized by the *J*-integral measurement based on the energy release rate interpretation of *J*. The critical *J* value (J_c) is shown to be a fracture characterizing parameter for the onset of the crack initiation in the injection moulded short glass fibre reinforced composite material. The J_c value of the composite is estimated by be 6.0 kJ m^{-2} . This value is in good agreement with the linear elastic strain energy release rate (G_c), since the composite exhibited a fairly linear stress-strain relationship. The estimated ratios of J_c to the total energy absorbed per unit uncracked area are in good agreement with the analytically obtained values after the remote energy dissipation due to fibre and matrix interaction away from the crack tip has been subtracted from the total energy.

Nomenclature

- J J-integral
- $J_{\rm c}$ The critical J
- G Elastic strain energy release rate
- $G_{\rm c}$ The critical G
- $K_{\rm I}$ Opening mode stress intensity factor
- K_{l_c} The critical K_l
- P Applied load
- x Load-point displacement
- B Specimen thickness
- E Young's modulus
- v Poisson's ratio

1. Introduction

There are some difficulties in characterizing fracture behaviour in short fibre reinforced composite materials. The difficulties stem from the heterogeneous nature of the crack tip region and the energy absorption at the fibre matrix interface remote from the crack tip region. Recently, Agarwal and others [1, 2] have attempted to circumvent these difficulties by utilizing the pathindependent J-integral proposed by Rice [3]. The Jintegral value based on its energy release rate interpretation [4, 5] was determined through the extrapolation of the strain energy to zero specimen length, and they considered this J-integral value as a fracture characterizing parameter. However, it has been shown that the extrapolation of the strain energy to zero specimen length does not partition the nonessential energy absorption away from the crack tip region [6]. One way of partitioning the nonessential energy is to integrate the fracture energy along the crack initiation locus line on the load-displacement record obtained from the identical specimens, which differ only in the initial crack size [7, 8].

- F Force
- Y Central deflection
- a/W Ratio of the crack length to the specimen width
- $\sigma_{\rm Y}$ Yield stress
- $U_{\rm t}$ Total strain energy in loading a specimen
- U_d Remotely dissipated strain energy after unloading
- $U_{\rm t-d} = U_{\rm t} U_{\rm d}$
- ϕ_{t} Ratio of J_{c} to U_{t} per unit uncracked ligament
- ϕ_{t-d} Ratio of J_c to U_{t-d} per unit uncracked ligament.

The purpose of this report is to present the results of the *J*-integral value obtained from short glass fibre reinforced thermoplastic and show that fracture initiation of the composite can be characterized by the *J*-integral.

2. Analytical background

Based on the energy release rate interpretation of J, the value of J-integral can be determined from the load-displacement curves as follows [4, 5, 9]:

$$J = -\frac{\partial U}{B\partial a}\Big|_{\text{constant displacement}}$$
(1)

where $U = \int P \, dx$, x is the load-point displacement, P is the applied load, and B is the specimen thickness. From an experimental point of view, however, J can not be determined easily when the critical displacement at which the fracture initiation occurs varies as a/W changes. In such a case, J can be evaluated along the locus of the crack initiation point [7, 8] which yields a linear relationship as follows.



Figure 1 Schematic load-displacement records of identical specimens which differ only in initial crack length. Points A, B, C, D indicate crack initiation, and dotted line indicates the locus line of the crack initiation points. 'OAD' = $J_c B$ ($a_4 - a_1$). Crack initiation locus $a_1 < a_2 < a_3 < a_4$.

$$J = -\frac{\Delta Uc}{\Delta a}\Big|_{\rm locus} \tag{2}$$

 J_c can be graphically represented on a loaddisplacement record schematically shown in Fig. 1. J_c is equal to area OAD divided by the ligament area $B(a_4 - a_1)$. This J is equivalent to G for the linear elastic materials [10]:

$$J_{\rm e} = G = (1 - v^2) \frac{K_{\rm l}}{E}$$
 (3)

where G is elastic strain energy release rate, $K_{\rm I}$ is opening mode stress intensity factor, v is Poisson's ratio, and E is Young's modulus. Srawley [11] has proposed a wide range stress intensity expression for the three-point bend specimen:

30% by weight, reinforced Polybutylene terephthalate (PBT), a semicrystalline thermoplastic polyester. The average length of glass fibre before the moulding was said to be 3.2 mm. The commercial grade of the resin, Valox 420, was supplied by the General Electric Company. The resins were vacuum-dried at 100° C for 2.5 h and moulded with the mould temperature of $30 \pm 1^{\circ}$ C and the barrel temperature of 295° C. The dimensions of the moulded specimens were 5.2 mm, 19 mm, and 165 mm for thickness, width, and length, respectively.

The initial notches were machined using double sided angle (45°) cutter whose surface was coated with cobalt to improve wear resistance against the glass fibre. Final cracks were made with sharp razors for industrial use and the once-used part of razor blade was not used again. The depth of razor notch was 0.5 mm. The razor notched specimens were on the shelf for one week to minimize the possible buildup of compressive residual stress caused by introducing a sharp crack.

Three-point bending tests were performed on Instron at a displacement controlled mode. Both the loading and unloading rate was 1 mm min^{-1} in terms of cross head speed. Room temperature and relative humidity were 28°C and 65% respectively. The span versus width ratio was 4. Loads and their corresponding displacements were recorded on a strip chart recorder. The start of whitening at the crack tip on the edge of the specimen was observed and this point considered as the crack initiation point. The crack initiation points were recorded on the load–displacement record during loading.

4. Results and discussion

Typical load-displacement curves for razor notched

$$K = \frac{PS}{BW^{3/2}} \frac{3(a/W)^{1/2} [1.99 - a/W(1 - a/W)(2.15 - 3.93a/W + 2.7a^2/W^2)]}{2(1 + 2a/W)(1 - a/W)^{1/2}}$$
(4)

Equation 4 is accurate within 0.5% over the entire range of a/w. The Young's modulus can be calculated based on the deflection of the slender beam including shear component [12] as follows:

$$E = \frac{SF}{BWY} \left[\frac{S^2}{4W^2} + 0.6(1+v) \right]$$
 (5)

where F is a force, S is the span of the three-point bending test, Y is the central deflection, and W and Bare the width and the thickness of the specimen.

The relation of J_1 to work done per unit uncracked area has been studied by Srawley [13]. The general relation is shown as below.

$$J_{\rm c} = \frac{\partial \ln U_{\rm c}}{\partial \ln (W-a)} \frac{U_{\rm c}}{B(W-a)} = \phi_{\rm t} \frac{U_{\rm c}}{B(W-a)}$$
(6)

Since the value of ϕ_1 relates the essential energy needed for the crack initiation to the total energy absorption, the ϕ_1 parameter can be useful for a singlespecimen-test method in evaluating J_c .

3. Experimental details

Specimens were injection moulded from glass fibre,

and machine notched specimens are shown in Fig. 2 and Fig. 3, respectively. The machine notched specimen showed more scatters in its J_c value than the razor notched one did. The effect of plastic deformation due to razor notch on J_c is not considered here although this may increase the value of J_c . What is emphasized in this report is a method of fracture characterization of short fibre reinforced materials and not the absolute value of J_c .

The remote energy dissipation due to fibre and matrix interaction was estimated by loading uncracked specimens to the corresponding initiation load and unloading after that. The loading and unloading curves of the uncracked specimens are shown in Fig. 4. This remotely dissipated energy for each a/W is plotted in Fig. 5. Although Agarwal *et al.* [1, 2] have reported that there is a critical a/W ratio above which the J value is independent of crack length, Fig. 5 indicates that a practical a/W ratio above which the remotely dissipated energy vanishes can not be found. When the a/W ratio is greater than 0.4 the amount of the dissipated energy does appear to be decreasing linearly with a/W.

The strain energy absorbed up to the crack initiation



Figure 2 Typical load-displacement curves for razor notched threepoint bend specimens. The dotted points represent the crack initiation.

point for each A/W ratio is shown in Fig. 6. The strain energy is determined from the loading curves and the crack initiation locus line. Detail of the locus method can be found elsewhere [7, 8]. The absolute value of this strain energy may differ from that shown in Fig. 6. However, the value of J is not affected as mentioned in an earlier report [6]. The critical J value (J_c) obtained from the slope of a least square fitted line in Fig. 6 is 6.0 kJ m^{-2} . This J_c is independent of initial crack sizes and specimen lengths since the remotely dissipated energy has been subtracted from the total energy absorbed [6]. If the J_c is to be a parameter which characterizes the fracture initiation then it should be independent of the specimen geometry so that it can be a genuine material property.

Plane strain condition must prevail at the onset of crack initiation in order for J_c to be independent of the specimen thickness. ASTM E 813-81 recommends $B > 25 J_c \sigma_Y^{-1}$, so the specimen thickness must be greater than 1.25 mm. Thus, the thickness of the composite for present study which is 5.2 mm satisfies the minimum thickness requirement.

The critical stress intensity factor (K_c) for each initial crack size has been calculated using Equation 4 and is shown in Fig. 7. The *E* value of 4.0 G Pa which is obtained experimentally by use of Equation 5 is



Figure 3 Typical load-displacement curves for machine notched three-point bend specimens. The dotted points represent the crack initiation.

used in Equation 3 to determine the G_c value which is shown in Fig. 8. K_{l_c} has been calculated at the observed initiation point not at the point recommended by ASTM. Although G_c can never be greater than J_c , some of the G_c values are shown to be higher. Nevertheless, G_c is shown to be independent of a/W as depicted by a fairly linear fit in Fig. 8. The average G_c is shown to be 5.8 kJ m⁻² and the standard deviation is 0.65 kJ m⁻². This value is in good agreement with J_c obtained directly from the load-displacement record, since the composite exhibited a fairly linear stressstrain relationship up to the crack initiation point.

The ratio of U_c to U_t and U_{t-d} are obtained from Equation 6 and plotted for each initial crack size as shown in Fig. 9 and Fig. 10 respectively. Since U_d does not contribute to the fracture process it must be subtracted from U_t when the relation of J_c to the work done per unit uncracked area is considered. It is shown that ϕ_{t-d} can be thought to be about 2 for a/W > 0.5. Srawley [13] reported that ϕ_t is practically constant at 2.02 \pm 0.02 for a/W which is greater than about 0.5 in the case of an ASTM E-399 three-point bend specimen of a linear elastic material. But his report is reportedly [12] based on the Poisson's ratio of 0.229. Hence our result of ϕ_{t-d} which has been obtained from the material of v = 0.42 does not show



Figure 4 The loading and unloading curves of initially uncracked specimens. The surrounded part represents the remotely dissipated energy for each corresponding specimen having initial crack length a_0 .



Figure 5 The remotely dissipated energy $(J m^{-1})$ versus a/W.

very good agreement with Srawley's report, especially when a/W is less than 0.5. After the Srawley's expression of ϕ_t is adjusted to include the influence of changing v, the values of ϕ_{t-d} show good agreement with the analytical expression of ϕ_t over the entire range of our experiment. The curves of Fig. 9 and Fig. 10 are plotted using the results which have been obtained by Kim and Joe [12]. In general, ϕ_t is dependent on both initial crack size and Poisson's ratio in



Figure 6 U_c /unit thickness against a/W graph. The slope of the straight line through the data points is the negative value of J_c .



Figure 7 The critical stress intensity factor (K_c) for each initial crack size. Total mean is 5300 (kJ m^{-3/2}) and the standard deviation is 4.6%.

materials exhibiting linear elastic behaviour under both plane strain and plane stress conditions [12].

Note that the specimen has been injection moulded, and therefore J_c should be dependent upon the moulding process parameters. For example, the resin melt



Figure 8 G_c versus a/W. G_c has been determined from K_c of Fig. 7.



Figure 9 The ratio of U_c to U_t for each initial crack size. The dotted line represents $\phi = 2$ and the curve represents $\phi_t(a/W)$ [12].

temperature, mould temperature, injection speed, and packing pressure may have a pronounced effect on the amount of crystallinity and residual stresses, and molecular orientation which in turn will affect J_c . In this report, we have shown that J_c can be considered as a parameter which characterizes the onset of crack growth in short glass fibre reinforced thermoplastic polyester. Therefore, one can study the effects of moulding process parameters on J_c of fibre composite materials.

5. Conclusions

Fracture initiation of short glass fibre composite has been characterized based on the *J*-integral measurements from the load-displacement records. The J_c value for 5.2 mm thick injection moulded short glass fibre reinforced thermoplastic polyester (PBT) is found to be 6.0 kJ m^{-2} . This value is obtained by eliminating the effect of the remote energy dissipation, thus is should be independent of the initial crack size and the specimen length. This value is in good agreement with G_c obtained from the stress intensity factor. The value of ϕ_t has been obtained experimentally and found to be in good agreement with the result of theoretical analysis when the dissipated energy due to the interaction between fibres and matrix has been subtracted from the total energy.



Figure 10 The ratio of U_c to U_{t-d} for each initial crack size. The dotted line represents $\phi = 2$ and the curve represents $\phi_1(a/W)$ [12].

Acknowledgement

This work was made possible by the financial support from the SKC Ltd through the SKC America Inc. The authors would like to thank J. S. Choi and H. Park for their interest in this work.

References

- 1. B. D. AGARWAL, B. S. PASTRO and P. KUMAR, Eng. Fracture Mechanics 19 (1984) 675.
- 2. B. D. AGARWAL, P. KUMAR and S. K. KHANNA, Compos. sci. tech. 25 (1986) 311.
- 3. J. R. RICE, J. Appl. Mech. 35 (1968) 379.
- J. A. BEGLEY and J. D. LANDES, ASTM STP 514 (American Society for Testing and Materials, Philadelphia, 1972) pp. 1-20.
- 5. J. D. LANDES and J. A. BEGLEY, ibid. pp. 24-39.
- 6. B. H. KIM and C. R. JOE, Eng. Fracture Mechanics 30(4) (1988) 493.
- 7. Idem, Polymer testing, 7 (1987) 355.
- 8. Idem, Int. J. Fracture 34 (1987) R57.
- J. R. RICE, P. C. PARIS and J. G. MERKLE, ASTM STP 536 (1973) pp. 231–245.
- J. D. G. SUMPTER and C. E. TURNER, ASTM STP 601 (1976) pp. 3–18.
- 11. J. E. SRAWLEY, Int. J. Fracture 12 (1976) 475.
- 12. B. H. KIM and C. H. JOE, Eng. Fracture Mechanics, in press.
- 13. J. E. SRAWLEY, Int. J. Fracture 12 (1976) pp. 470-474.

Received 10 February and accepted 13 June 1988